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A theoretical analysis of extensive air showers
II. Fluctuations of muon and electron numbers

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Abstract. The results reported form a continuation of the analysis given by de Beer *et al.* in 1966. In the present work an examination is made of the fluctuations expected in the electron and muon components from shower to shower, with particular regard to the number, lateral distribution and height of origin of the muons.

It is shown that fluctuations can affect considerably the picture of showers drawn from results with practical air shower arrays, largely through the correlations that exist between the various components.

The important problem of deriving results of significance to high-energy nuclear physics and to astrophysics is studied, and it is shown that the relative fluctuation of the muon number in showers of fixed size has some sensitivity to the characteristics of high-energy interactions, whereas the relative fluctuation of the shower size for constant muon number is sensitive to the mass composition of the primary particles.

1. Introduction

The results reported here form part of a wider programme of calculations on the development of extensive air showers, the first part of which has been published already by de Beer *et al.* (1966, to be referred to as I). A brief report of part of the present work has been given by the authors (de Beer *et al.* 1967).

The purpose of the programme is to devise theoretical models which will explain the various features of extensive air shower data, and from them to derive information concerning astrophysics and the character of ultra-high-energy interactions.

As a basis for initial calculation, a model has been used for the interaction of nucleons and pions in the atmosphere, the details of which are given in I. An important feature is the use of the so-called CKP model (Cocconi, Koester and Perkins 1961) for the energy spectrum of pions for nucleon–light-nucleus interactions, this model having been derived from the experimental data in the machine region. An extrapolation of the model to the much higher energies which are involved in extensive air showers may not, of course, be valid, but by analysing the differences between the theoretical predictions and experimental observations one can hope to derive a more realistic model.

The present paper refers to calculations concerning sea-level extensive air showers, the main problem being that of the fluctuations in the muon and electron components. One of the aims is to derive the magnitude of the fluctuations in the N_μ/N_e ratio and to examine the influence of the fluctuation effects on the picture of showers recorded in various experiments. The main purpose of the fluctuation analysis is to find how the width of fluctuations is affected by changes in the high-energy interaction model and in the primary composition. This problem is of contemporary interest because a number of measurements on the width of the fluctuations have been reported during the last few years, the most recent being by Vernov *et al.* (1967) and Gawin *et al.* (1967). Some aspects of these problems have been studied by Dedenko (1964) and Khristiansen *et al.* (1966 a), but the present work is more comprehensive.

The sequence of the paper is as follows. The next section gives the results of calculations on the fluctuations in proton and heavy-nucleus-initiated showers of fixed primary energy.

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This is followed by an examination of the heights of origin of the muons, after which the dependence on shower size of the width of the distribution in muon number is examined for showers of a given electron number and that in electron number for showers of a given muon number. Finally, some effects of fluctuations on observed shower parameters are discussed.

The problem of comparison with existing experimental data is reported separately in the following paper (Adcock *et al.* 1968).

2. Fluctuations in electron and muon numbers and in the lateral distribution of muons

2.1. *Fluctuations in total number of electrons*

Calculations on fluctuations in total numbers of electrons and muons, as well as in the lateral distribution of muons, have been made using the model of extensive air shower development described in I. The calculations are based mainly on the Monte Carlo method, and give the possibility of calculating not only the fluctuations of the separate parameters but also the correlations between them. Although the CKP model was a basic feature, some calculations have also been made using a simpler model, in which it is assumed that half the secondary pions are emitted in the forward direction in the C system and that they share equally the available energy. The model is termed the CE model and it was used mainly to study the sensitivity of the results to the assumed model. It is intended to deal later with more realistic alternative models involving isobar production.

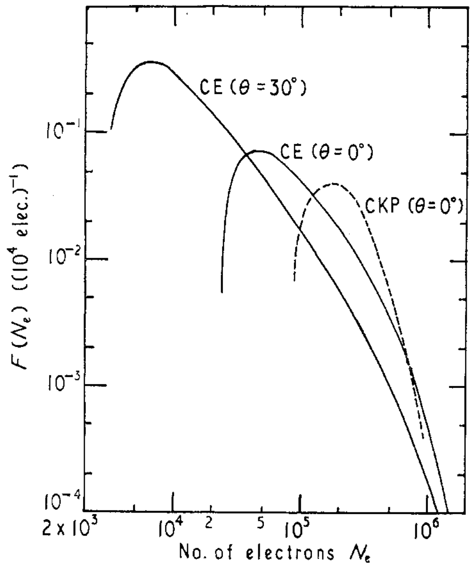

Figure 1. The distribution in N_e at sea level for primary protons of $1{\cdot}56 \times 10^6$ GeV.

The results on the frequency distribution of N_e for CKP and CE models are shown in figure 1, where results are given for $\theta = 0°$ and 30°. The figure shows that the width of the distribution is sensitive to the adopted model, a fact that raises hopes of distinguishing between the various models experimentally. The enhanced width is also apparent from the extensive calculations reported by Khristiansen *et al.* (1966 b) for the case of a similar model. It is also clear from the figure that there is some sensitivity to zenith angle, a fact which indicates that a knowledge of shower direction is important in experimental observations.

The relative contributions to the width arising from fluctuations in inelasticity and in the positions of the interaction points have been calculated, and it is found that the latter is substantially more important.

Following previous work, we use the ratio $\sigma_{N_e}/\bar{N}_e$ as a quantitative estimate of the width. Here σ_{N_e} is the standard deviation of the N_e distribution and $\bar{N}_e$ its average value. The variation of width with primary energy has been examined, and it is found that for the CKP model the following expression is valid:

$$\sigma_{N_e}/\bar{N}_e = 7{\cdot}0\, E_p^{-0{\cdot}15} \tag{1}$$

where E_p is the primary energy expressed in GeV.

2.2. *Fluctuations in total number of muons*

As was mentioned in I, fluctuations in the total number of muons in showers of fixed primary energy are small when the CKP model is used. The fluctuations for the CE model are also found to be small. An interesting feature is the clear correlation between the total number of muons in a shower and the corresponding total number of electrons. The correlation is positive when low-energy muons are considered (1 GeV threshold), and negative for high-energy muons (100 GeV threshold), as can be seen in figure 2.

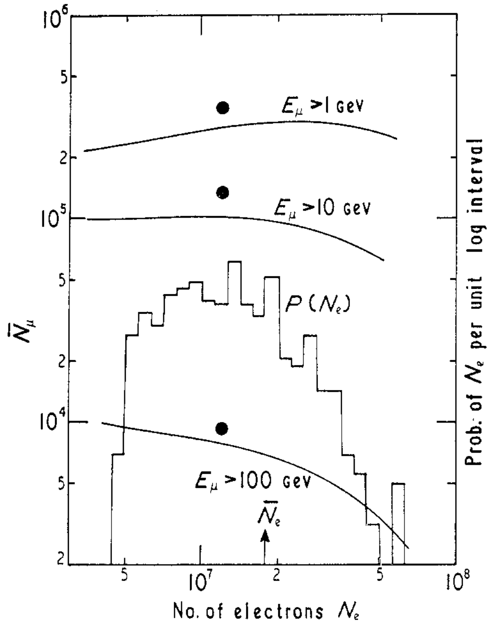

Figure 2. The dependence of N_μ on N_e for showers produced by protons of 10^8 GeV. The full circles refer to heavy primaries, $A = 14$.

If we neglect the biggest showers, the dependence for muons above 1 GeV in showers of constant primary energy is

$$N_\mu(E_p = \text{const.}) \propto N_e^{0{\cdot}15} \tag{2}$$

The effects follow from the fact that for high-energy muons only the probability of pion decay is significant, whereas for low-energy ones their survival probability against μ–e decay is the important feature. The smaller showers, for fixed primary energy, are produced on average higher, so they contain more high-energy muons and fewer low-energy muons.

2.3. *Fluctuations in the muon lateral distribution*

Fluctuations in the muon lateral distribution are relatively wide. Examples of the lateral distributions have already been reported by the authors (de Beer *et al.* 1967). Briefly, it was shown that if the fluctuations are neglected then the theoretical estimate of the muon density is too low at large distances from the shower core.

Another effect which is relatively striking is the negative correlation between the width of the muon lateral distribution and N_e. The effect increases somewhat as the muon threshold energy increases. It can be understood in terms of showers which develop late, having large N_e, and, by virtue of low muon origins, narrow lateral distributions for the muon component. The effect is shown quantitatively in figure 3, where the dependence of $\bar{r}_\mu$, the mean distance from the shower axis, on N_e is shown for showers of 10^8 GeV and for various muon threshold energies.

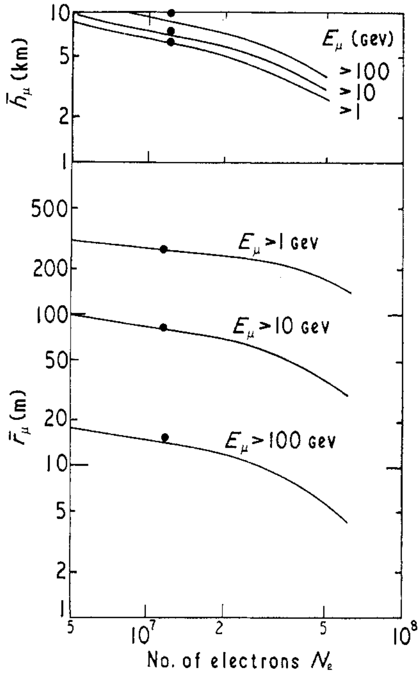

Figure 3. The mean radial distance and mean height of origin of muons from proton primaries of 10^8 GeV. The full circles refer to heavy primaries, $A = 14$.

2.4. *Mean-value dependences and the characteristics of showers initiated by heavy primaries*

2.4.1. *Proton-initiated showers.* The dependences of the mean number of muons $\bar{N}_\mu$ and mean radius $\bar{r}_\mu$ on E_p have been calculated with muon energy as parameter. The relation between $\bar{N}_\mu$ and E_p is

$$\bar{N}_\mu = BE_p{}^\alpha \text{ for protons} \tag{3}$$

where B and α are constants dependent upon threshold energy. Their values are given in table 1. The dependence of $\bar{r}_\mu$ on primary energy is shown in figure 4. The dependence

Table 1. Coefficients in the relation $N_\mu = BE_p{}^\alpha$

E_μ threshold (GeV)	1	3	10	30	100
α	0·9	0·89	0·88	0·87	0·86
B	0·019	0·014	0·0091	0·0039	0·0010

for muons with energy above 1 GeV can be expressed analytically by the function

$$\bar{r}_\mu = 295 - 8{\cdot}3\,(\log E_p - 4) \tag{4}$$

where E_p is the primary energy, measured in GeV, and $\bar{r}_\mu$ is in metres.

Finally, from our calculations the total number of electrons varies with primary energy as

$$\bar{N}_e = kE_p{}^{1\cdot15}. \tag{5}$$

The proportionality coefficient k is sensitive to the model adopted; for the CKP model $k = 6{\cdot}3 \times 10^{-3}$. (In all the calculations model II, referred to in I, has been used.)

2.4.2. *Heavy-nucleus-initiated showers.* The results shown so far refer to proton-initiated showers; some calculations have also been made for showers initiated by heavy primaries. In view of the lack of detailed knowledge of the fragmentation probabilities it has been assumed, following I, that the heavy-initiated showers can be considered as a superposition

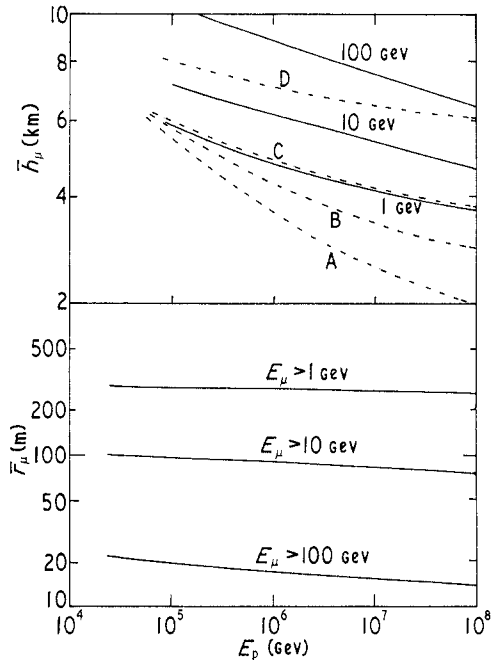

Figure 4. The mean radius and mean height of origin of muons as a function of primary energy, with muon threshold energy as parameter. The broken curves refer to muons with $E_\mu > 1$ GeV falling within various radial distances: A, $r < 3{\cdot}16$ m; B, $10 < r < 31{\cdot}6$ m; C, $100 < r < 316$ m; D, $r > 1000$ m.

of A showers, each with A times smaller energy, A being the mass number of the primary nucleus. Such an approach seems to be justified since fragmentation probabilities appear to be relatively high, and, furthermore, the contribution to the shower picture at the observation level from the first interaction, which may occur as an interaction of the bound nucleus, is relatively small.

For the adopted model the width of the fluctuations in heavy-initiated showers is given by

$$\left(\frac{\sigma}{\bar{N}_e}\right)_A = \left(\frac{\sigma}{\bar{N}_e}\right)_p A^{-1/2} \tag{6}$$

where the energy per nucleon is the same for the two nuclei.

The average parameters of the showers can be obtained using the expressions giving the energy dependencies of the average values of the parameters in proton-initiated showers. The average values in the case of a nitrogen-initiated shower ($A = 14$), calculated using expressions (3) to (5), are marked on figures 2 and 3. It can be seen that the mean radius is near to that in proton-initiated showers of the same energy and size, whereas the total number of muons is higher in showers of the same primary energy. The last effect follows from the fact that α in expression (3) is smaller than 1.

3. Height of origin of the muons

Another parameter of the muon component which can be considered as measurable is their mean height of origin. Besides the mean height, other related parameters are interesting; these are as follows: the distribution of height around the mean value, fluctuations of the height from shower to shower and its dependence on distance from the core.

Some of these parameters have been calculated and the results are presented below. The curves in figure 4 give the dependence of the average height on primary energy for different muon energy thresholds. The character of the curves can be explained from qualitative considerations: muons with higher energies should be produced higher up since their parent pions require a lower density of atmosphere for decaying. In smaller showers the muons are expected to be produced higher since the maximum of development of such showers is situated higher. In figure 3 is shown the variation of the average height with the number of particles in the shower for proton-initiated showers of fixed primary energy. The full circles show the mean values of the height, and corresponding mean values of N_e, for showers initiated by primary nuclei with $A = 14$.

It can be seen that in showers of the same size from primaries of the same energy the height is not very strongly dependent on the mass of the primary particle. However, for the practical case of a spectrum of primary energies, showers of the same size will, in general, have primary energy dependent on the mass of the primary particle. Thus the mean height of origin of the muons will depend somewhat on the mass of the primary particle. Another feature of the curves in figure 3 is the expected strong negative correlation between N_e and the height.

4. Sensitivity of the fluctuations in the N_μ/N_e ratio to the mass composition of the primary particles

4.1. *The adopted primary spectra*

An estimate of the sensitivity of the fluctuations in muon and electron numbers to the mass composition of the primaries has been made by performing calculations for the following alternative mass spectra:

(i) Protons only.

(ii) A constant composition throughout, consistent with that found in the region of a few GeV, denoted by spectrum A.

(iii) A variable composition with enhanced contribution from heavy nuclei above about 10^{15} ev and reappearance of protons from extragalactic sources above 10^{17} ev. This case, denoted by spectrum B, has been taken following indications from a number of experiments (which are, however, not conclusive). It is assumed that the galactic primary spectrum is modulated by a factor $\exp(-R/R_0)$, where the rigidity $R_0 \simeq 3\times10^{15}$ ev. The factor for the extragalactic component is taken as $1-\exp(-R/R_0)$.

In all cases the energy spectra are designed to give the size spectrum of near-vertical extensive air showers as measured experimentally at sea level. The mass composition of the primary particles in the region below 10^{15} ev which has been assumed in the present paper is given in table 2.

Table 2. The adopted primary mass composition below 10^{15} ev

Group of nuclei	P	α	L	M	H
A	1	4	10	14	31
Proportion	1	0·5	0·05	0·18	0·23

The proportions refer to particles of the same total energy.

The fluctuation distributions and their average values and widths for groups of showers initiated by different groups of primary nuclei have been obtained from the corresponding values for the proton-initiated showers. The widths of the fluctuations have been calculated according to equation (1), and the average values using equations (3) and (5), α in equation (3) having been taken as 0·9. Because α does not depend strongly on muon energy the results obtained can be used for comparison with experimental results on N_μ/N_e fluctuations obtained for different muon threshold energies. In the calculations the N_μ–N_e correlation shown on figure 2 has been neglected, at least up to 10 GeV threshold.

Fluctuations in the electron and muon components have been studied experimentally in two ways. Firstly, showers of constant number of muons have been selected, the number of muons being derived from density measurements at very few points, and the frequency distribution of the corresponding electron sizes has been recorded. Secondly, the frequency distribution of the number of muons has been measured for showers of the same electron size. It is therefore necessary to calculate theoretically the distributions to be expected under these respective conditions.

4.2. *The distribution of N_e for fixed N_μ*

In view of the narrowness of the distribution of N_μ for fixed E_p (see, for example, the distributions in I), showers of fixed N_μ correspond to primaries of almost unique energy for each primary mass. The derivation of $\sigma_{N_e}/\bar{N}_e$ for each primary spectrum is straightforward, and the results for showers in the vertical direction, using the CKP and CE relations, are shown in figure 5.

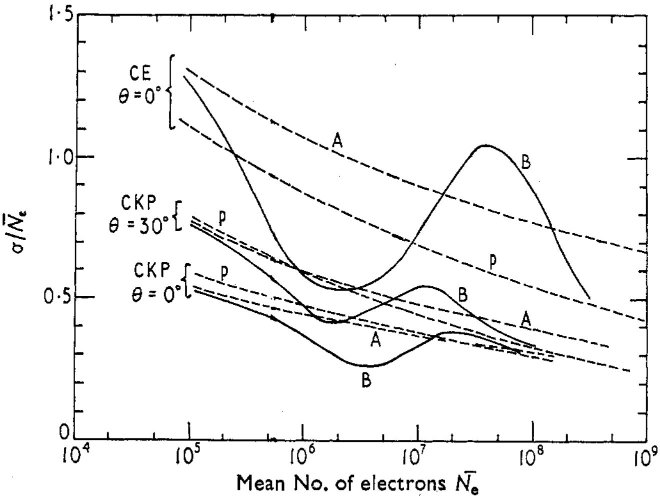

Figure 5. Relative standard deviation of N_e for fixed N_μ. The symbols are defined in the text. The abscissa is the mean electron size corresponding to the particular value of N_μ.

The oscillation of $\sigma_{N_e}/\bar{N}_e$, which is such a pronounced feature of the variation for the modulated spectrum, arises as follows. The minimum is due to the rapid transition from protons to heavy nuclei after modulation starts. The maximum comes from the fact that in the size range 10^7–10^8 there are comparable numbers of heavy nuclei and extragalactic protons contributing, the particle energies and consequent mean electron sizes being rather different in the two cases.

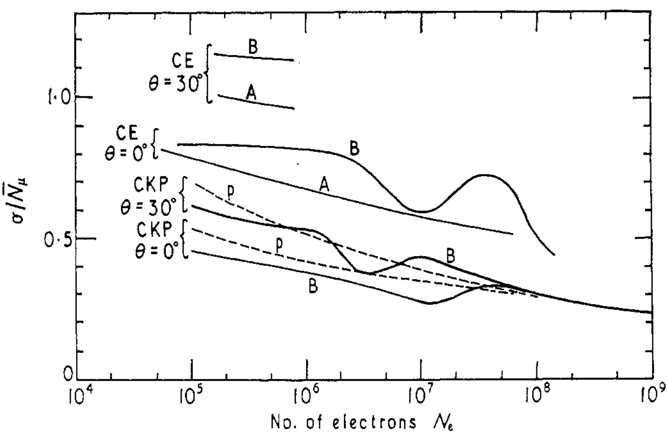

Figure 6. Relative standard deviation of N_μ for fixed N_e. The symbols are defined in the text.

There is clearly some sensitivity to both the mass spectrum and the form of the interaction model; a consideration of actual experimental results is given in the following paper.

4.3. *The distribution in N_μ for fixed N_e*

In this case the calculation of the relative standard deviations of the muon distribution is more difficult because it involves deriving the primary energy spectrum of each mass component from its inverse—the distribution in N_e for fixed E_p.

The relative standard deviations of the muon distributions have been calculated for fixed values of N_e and for the alternative mass spectra, and the results are given in figure 6. The most important feature of the results is that the curves corresponding to the various mass spectra are so close as to be quite indistinguishable by experiment. On the other hand, there is appreciable sensitivity to the interaction model.

5. Influence of the fluctuations in various parameters on the picture of showers recorded in experiments

5.1. *Change of the shower picture by the constant-size requirement*

The effects of fluctuations are important when a comparison of theoretical calculations with experimental data is made, because the calculations are usually made for showers of fixed primary energy, whereas the experimentally observed showers are grouped according to some observable quantity (usually size) which is not related uniquely to primary energy.

Showers observed under the condition of fixed size are mainly those which have a relatively high ratio of N_e/E_p. This high ratio is caused largely by the fact that the main development of the showers took place relatively lower in the atmosphere. Some qualitative proof of this point comes from the dependence of the muon height of origin on N_e for fixed E_p shown in figure 3.

As a result of this effect, the showers recorded under the condition of fixed size have a lower N_μ/N_e ratio and the muons have a narrower lateral distribution in comparison with showers of fixed primary energy. Some quantitative estimations of the effect are given in table 3, which shows the factor of increase of the $\bar{N}_e/E_p$ ratio, due to recording under the condition of fixed size, as a function of a shower size. A comparison of the muon lateral distribution in showers of fixed size and fixed primary energy, for energies in the region of 10^{15} ev, shows that the mean radius is reduced by 11% (when a large radial distance cut-off is applied at 1000 m).

Table 3. Factor by which the $\bar{N}_e/E_p$ ratio is increased when recording under fixed-size condition

Size	$4\cdot5\times10^3$	$8\cdot2\times10^4$	$1\cdot6\times10^6$	$2\cdot8\times10^7$
Factor	4·65	2·57	1·96	1·62

5.2. *Effect of the correlations in $\bar{r}_\mu$ and N_e*

Another effect which is due to fluctuations in the muon lateral distribution is shown in figure 7. In this figure are plotted distributions of N_e, N_e/N_μ and $N_e\bar{r}_\mu^2/N_\mu$ for two muon energy thresholds. (The quantity $N_\mu/\bar{r}_\mu^2$ can be considered as proportional to the muon density Δ_c in the central part of the shower).

From this figure it can be seen that fluctuations of the Δ_c/N_e ratio observed at small distances from the shower core are smaller than the fluctuations in N_μ/N_e and N_e. The effect is due to the negative correlations between N_e and $\bar{r}_\mu$ shown in figure 3.

Using data from the present paper, an estimate has been made of the change of width of the N_μ/N_e distribution with distance from the shower axis of the muon detector whose recorded muon number is used to give N_μ. The dependences for 1 and 10 Gev threshold are given in figure 8. The results for the case of 1 Gev threshold were calculated, taking into account the positive correlations N_μ–N_e for fixed E_p presented in figure 2 as well as the negative correlations from figure 3.

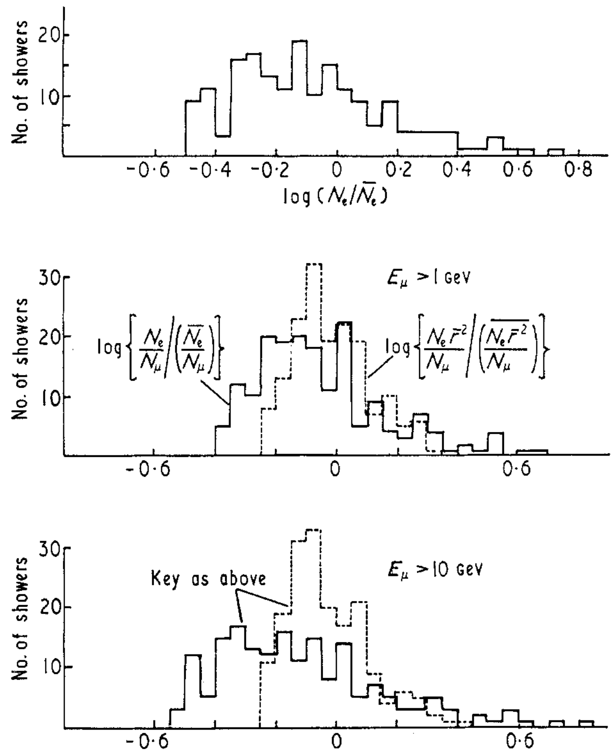

Figure 7. The distribution of N_e, N_e/N_μ and $N_e\bar{r}^2/N_\mu$ for showers produced by primary protons of 10^7 GeV incident at 30°.

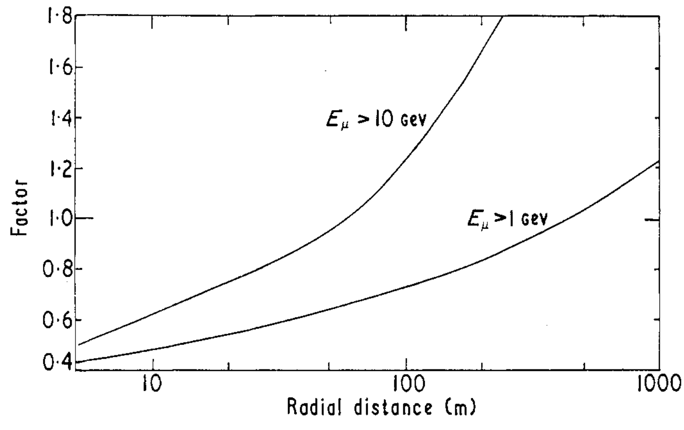

Figure 8. The factor by which the true width of the N_μ/N_e distribution is multiplied as a function of the radial distance from the core of the point at which the muons are recorded.

6. Conclusions

(i) If we apply the CKP model to showers initiated by protons, fluctuations in the muon lateral distribution are found to be large, although fluctuations in the total number of muons are relatively small.

It should be stressed, however, that the results obtained should be considered as a lower limit to the effects expected to exist in real extensive air showers, because only fluctuations in total inelasticity and in the positions of the leading particles have been taken into account. The present model is probably sufficiently good for calculations of the fluctuations in total number of particles (mainly electrons), but there exist some effects, such as fluctuations in the energy transferred to the electromagnetic component and fluctuations in multiplicity, which can seriously increase fluctuations in both the lateral distribution of the muons and their total number.

(ii) In the interpretation of the results of extensive air shower experiments the effect of fluctuations needs careful consideration, the effect of correlations being particularly important. An example is the correlation of electron size and mean muon radius in showers of fixed primary energy.

(iii) In order to study the primary composition, measurements of the fluctuations of N_e for fixed N_μ could be used. The experimental effort should be concentrated on measurements of changes in the width of the fluctuations as a function of size rather than on finding an absolute value of it at any particular point. This is due to the fact that the absolute width of the fluctuations seems to be not very sensitive to primary composition, but, on the other hand, the shapes of the functions in figure 5 are different for different spectra. Relative measurements are clearly safer because some of the inevitable systematic errors will vary only slowly with size.

(iv) An investigation of the fluctuations in N_μ for fixed N_e gives information about the characteristics of high-energy collisions. The difference between the fluctuations expected for the model involving the CKP relation and that using the unrealistic CE relation is so large that there should still be significant sensitivity when more realistic models are used as alternatives, such as, for example, models involving isobar production. As a result of the relative insensitivity of the width to the primary composition one can hope to obtain, in this way, some information about the interaction model before the primary composition problem is solved.

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References

ADCOCK, C., *et al.*, 1968, *J. Phys.* A (*Proc. Phys. Soc.*), [2], **1**, 82–8.

DE BEER, J. F., *et al.*, 1967, *Proc.* 10*th Int. Conf. on Cosmic Rays, Calgary*, 1967 (Ottawa: National Research Council of Canada).

DE BEER, J. F., HOLYOAK, B., WDOWCZYK, J., and WOLFENDALE, A. W., 1966, *Proc. Phys. Soc.*, **89**, 567–85.

COCCONI, G., KOESTER, L. G., and PERKINS, D. H., 1961, *High Energy Physics Study Seminars*, No. 28, Pt 2, UCID-1444, 1–36 (Stanford, California: Lawrence Radiation Lab.).

DEDENKO, L. G., 1964, *Sov. Phys.–JETP*, **19**, 1251–67.

GAWIN, J., *et al.*, 1967, *Proc.* 10*th Int. Conf. on Cosmic Rays, Calgary*, 1967 (Ottawa: National Research Council of Canada).

KHRISTIANSEN, G. B., ATRASHKEVITCH, V. B., KALMYKOV, N. N., and FOMIN, Yu. A., 1966 a, *Proc.* 9*th Int. Conf. on Cosmic Rays, London*, 1965 (London: Institute of Physics and Physical Society), pp. 774–8.

KHRISTIANSEN, G. B., *et al.*, 1966 b, *Proc.* 9*th Int. Conf. on Cosmic Rays, London*, 1965 (London: Institute of Physics and Physical Society), pp. 799–801.

VERNOV, S. N., *et al.*, 1967, *Proc.* 10*th Int. Conf. on Cosmic Rays, Calgary*, 1967 (Ottawa: National Research Council of Canada).